

MANAGEMENT AND OPTIMIZATION MODELS OF RISK SITUATIONS IN ECONOMICS

O.A. Kovaleva, E.K. Rumyantsev

Represented by Doctor of Economics, Professor S.P. Spiridonov

Key words and phrases: optimization of risk situations; risk measure; statistical method; utility function.

Abstract: This paper presents the main classes of mathematical models to make management decisions for risk optimization. The application of statistical methods to quantify the risks associated with making financial decisions has been discussed.

Any sphere of human activity, especially economics and business is related to decision-making under conditions of lack of information, which in its turn can be caused by a variety of factors, both objective and subjective. It is quite common to make decisions in risky conditions: the effects of these decisions can be multiple and not always positive. Another problem is that it is not possible to predict these effects to full or some extent. A risk is understood as a likelihood of incurring losses or additional costs by an individual resulting from the implementation of certain financial policies. We shall consider the risks associated with making financial decisions.

Models that enable to make management decisions on optimization of risk situations are becoming of particular relevance. To meet these challenges, the most commonly used mathematical tools are still methods of probability theory.

Mathematical apparatus of risk optimization theory depends on the nature of the initial information available at the time of decision making and the chosen way of describing uncertainty. Currently, the most common are three classes of mathematical models describing the uncertainty: stochastic models, linguistic models, non-stochastic (game) models. In stochastic models uncertainty is described by probability distribution on a given set, in linguistic models it is described verbally by the membership function, in the non-stochastic (game) models a set of values is given. A necessary prerequisite for sustainable use of stochastic models is statistically significant information about past realizations of undefined variable. The expert judgment on the degree of susceptibility of an event's potential to being realized is used to construct membership function in linguistic models. In case of building non-stochastic (game) models a set of

Ковалева Ольга Александровна – кандидат технических наук, доцент кафедры «Экономика», e-mail: solomina-oa@yandex.ru; Румянцев Евгений Константинович – кандидат экономических наук, доцент кафедры «Экономический анализ и качество», ТамбГТУ, г. Тамбов.

values of the elementary events that could potentially be implemented is given. Thus, the transition from stochastic models through linguistic ones to non-stochastic models corresponds to the decrease in users' awareness of the modeled object.

Often decision-taking is guided by the size of the expected return. Meanwhile, for every decision there is a risk. In this respect, relation between risk and return is one of the most relevant problems to economic activity and, in particular, financial management.

Since the result of financial decision-making is generally not known in advance due to the uncertainty of market conditions, even in the near future, this result can be viewed as a random variable.

The ultimate results when using statistical method are as follows.

1. The average value X (arithmetic mean) of the random variable under study (the consequences of any action, such as costs, revenue, results, etc.). From the theory of statistics it is known that for a limited number n of possible values of a random variable, its mean value is determined by the expression

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n} = \frac{\sum_{i=1}^n X_i}{n},$$

where n is the number of units of the examined characteristic.

The average value is a generalized quantitative description of the expected result.

2. An important characteristic that determines the measure of variability of a possible result is dispersion defined by

$$\sigma^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n} = \bar{X}_i^2 - (\bar{X})^2.$$

3. As a measure of risk of financial decision we shall assume the standard deviation σ of the main indicator of the decision defined by $\sigma = \sqrt{\sigma^2}$.

4. In practice, the dimensionless quantity of risk is often used; it is the coefficient of variation V measured as a percentage. This indicator is defined as

$$V = \frac{\sigma}{\bar{X}}.$$

Under the same or comparable to average \bar{X} is normally taken the decision under which σ is smaller.

5. Since the achievement of the desired result (e.g., the amount of expenses) is influenced by many random factors, it is natural to be a random variable. One of the characteristics of a random variable X is the distribution of its probability. As practice shows, to characterize the distribution of socio-economic phenomena the so-called normal distribution is most commonly used.

Let's consider the application of this method on a specific example.

The investment company offers three projects with different degrees of risk and specified realizations of cash flows (Table).

Projects for Investment Companies

Characteristic	Project 1			Project 2			Project 3		
Probability of event	0.2	0.6	0.2	0.3	0.5	0.2	0.3	0.3	0.4
Expected return, m rubles	40	50	60	50	60	80	50	70	100

The task: Determine which project should be selected. We introduce an additional condition: the investment company has a debt to the bank in the amount of 70 m rubles, which requires redemption.

Utility is a number attributed to DM (decision maker) to each possible outcome. For example, the utility can be expressed as the amount of revenue or profit in some units. Each decision maker has a utility function, which shows his preference for one or another outcome depending on his attitude toward risk.

J. von Neumann and O. Morgenstern showed that decision maker will seek to maximize the expected utility. A measure of DM risk inclination depends, in part, on his financial position or the current conditions of decision-making i.e. this characteristic is not the same. In this example, we consider the expected return as an expected utility.

First, calculate the mean values, namely, the corresponding expectations of discrete random variables:

$$\bar{X}_1 = 0.2 \cdot 40 + 0.6 \cdot 50 + 0.2 \cdot 60 = 50 \text{ m rubles};$$

$$\bar{X}_2 = 61 \text{ m rubles}; \quad \bar{X}_3 = 76 \text{ m rubles}.$$

Followed we calculate the variance and the corresponding standard deviations, as well as the dimensionless risks.

$$\sigma^2(X_1) = 1600 \cdot 0.2 + 2500 \cdot 0.6 + 3600 \cdot 0.2 - 2500 = 40 \text{ m rubles},$$

$$\sigma(X_1) = 6.3, \quad V(X_1) = 12,6 \%;$$

$$\sigma^2(X_2) = 109 \text{ m rubles}, \quad \sigma(X_2) = 10.4, \quad V(X_2) = 17 \%;$$

$$\sigma^2(X_3) = 444 \text{ m rubles}, \quad \sigma(X_3) = 21, \quad V(X_3) = 27.7 \%.$$

As it can be seen from the above solution the first project has a minimal risk; also, the expected return is minimal for all three projects, 50 million rubles compared to 76 million rubles for the third project. With the increase in the expected return the risk increases, while the risk in the third project is twice as much as that of the first project. If the investor tends to be cautious, he will be inclined to accept the first project. Meanwhile, the investor's behavior is influenced by specific financial circumstances. So, with the additional condition the investor is likely to be seriously considering the participation in the second and third projects. In other words, the investor chooses the project according to their utility function.

Thus, the main difficulty in applying mathematical methods of analysis based on statistical description of the uncertainties to risk optimization results is

caused by possible inadequacy of the models and inaccurate identification of parameters. As a rule, the inadequacy of the models is the result of incomplete information about the object and sources of negative results of the optimization process.

References

1. Дубров, А.М. Моделирование рискованных ситуаций в экономике и бизнесе / А.М. Дубров, Б.А. Лагоша, Е.Ю. Хрусталева. – Изд. 2-е, перераб. и доп. – М. : Финансы и статистика, 2002. – 172 с.
2. Красс, М.С. Математика в экономике. Математические методы и модели : учебник / М.С. Красс, Б.П. Чупрынов. – М. : Финансы и статистика, 2007. – 544 с.

Модели управления и оптимизации рискованных ситуаций в экономике

О.А. Ковалева, Е.К. Румянцев

ФГБОУ ВПО «Тамбовский государственный технический университет», г. Тамбов

Ключевые слова и фразы: мера риска; оптимизация рискованных ситуаций; статистический метод; функция полезности.

Аннотация: Представлены основные классы математических моделей, позволяющих принимать управленческие решения по оптимизации рискованных ситуаций. Рассмотрено применение статистического метода для количественной оценки рисков, связанных с принятием финансовых решений.

© О.А. Ковалева, Е.К. Румянцев, 2013